Area Formulas
for Parallelograms, Triangles, and Trapezoids

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Welcome to the AIMS Essential Math Series! .................. 3

BIG IDEA: The area of a parallelogram is found by multiplying base by height. All parallelograms that have the same length base and height have the same area no matter how far they have been skewed.

Lesson One: Parallelogram Cut-Ups ........................................... 9

**Days** 1 and 2

**Investigation** Parallelogram Cut-Ups ..................................................... 10

By measuring the base, height, and sides of parallelograms, students recognize that the height and sides of parallelograms are different lengths. They learn to use the appropriate measures to determine the perimeters of parallelograms.

By cutting up a parallelogram and reforming it into a rectangle, students discover the relationship of the two and the similarity of finding the areas of both. A critical understanding is being able to differentiate between a side and the height. Students should come to recognize that any of the sides can be designated as the base.

**Comic** Parallelogram Cut-Ups .......................................................... 15

The comic summarizes the relationship of a parallelogram to a rectangle with the same base and height and develops the meaning of the general formula for area.

Lesson Two: Areas on Board: Parallelograms ....................... 17

**Day** 3

**Investigation** Areas on Board: Parallelograms ................................. 19

By recognizing that every parallelogram can be transformed into an equal area rectangle, students confirm the area formula of base times height.
A parallelogram is skewed while keeping its sides the same length resulting in a changed height and area. The parallelogram is then skewed again, this time changing the length of the sides to keep the height constant resulting in a constant area. The visual display accentuates the critical nature of the height.

These problems provide an assessment of understanding of the area formula as students are asked to find various unknown dimensions.

A triangle is half the area of a parallelogram with the same height and width. The formula $A = \frac{1}{2} (b \cdot h)$ describes this relationship.

By matching pairs of congruent triangles and forming parallelograms with them, students will recognize that a triangle is half of a parallelogram. This understanding connects with the formula for the area of a triangle as: $A = \frac{1}{2} (b \cdot h)$.

The comic emphasizes the relationship of two congruent triangles to a parallelogram and develops the formula by showing how it represents this relationship.

All triangles can be cut so their pieces can be reformed into parallelograms. A parallelogram will have a base or height that is half the base or height of the triangle from which it was made. The experience provides a visual model of the formula in the form $A = \frac{1}{2} b \cdot h = b \cdot \frac{1}{2} h$.

The comic emphasizes the relationship of two congruent triangles to a parallelogram and develops the formula by showing how it represents this relationship.
Lesson Five: Areas on Board: Triangles

A geoboard or dot paper provides a grid for counting square area. By looking at triangles with equal areas, students find that the triangles have a common base and height. Multiplying the base and height gives the area of a rectangle that is twice the size of the triangle.

Triangle Transformations

A triangle is transformed into a parallelogram or rectangle in four ways. The vivid visual models reinforce the understanding of the area formula and demonstrate several forms of the formula.

Practice: Bigger Triangles

Using larger triangles drawn on dot paper, students apply and practice what they learned from the initial investigation.

Problem Solving Triangles

Students use the area formula to solve problems with triangles.

BIG IDEA:

The area of a trapezoid is calculated by multiplying the average base by the height. The formula is

\[ A = \frac{1}{2} (b_1 + b_2) \cdot h \]

Lesson Six: Trapezoid Cut-Ups

Cutting out and combining two trapezoids into a parallelogram demonstrates the area formula

\[ A = \frac{1}{2} (b_1 + b_2) \cdot h \]

Any two congruent trapezoids form a parallelogram that has a base that is the combined lengths of the top and bottom bases of the trapezoid. Dividing the area of the parallelogram by two gives the area of the trapezoid.

The comic demonstrates that two congruent trapezoids form a parallelogram and reinforces how the formula represents this relationship.
Lesson Seven: Trapezoids to Parallelograms

Day 12

Investigation Trapezoids to Parallelograms

Cutting a trapezoid in half and rotating it forms a parallelogram of the same area. Calculating the area of the parallelogram, which is half the height of the trapezoid, gives the area of the trapezoid. The transformation of the trapezoid is a visual model of the formula in the form \( A = \frac{1}{2} h \cdot (b_1 + b_2) \).

Day 13

Animation Trapezoid Tumbles

The animation demonstrates both doubling the trapezoid to make a parallelogram twice as big as the trapezoid and cutting the trapezoid in half to make a parallelogram equal in size. The dynamic visual reinforces the ideas developed in the investigations and encourages students to seek the commonalities of the formulas to clarify the concepts expressed in the formulas.

Problem Solving Trapezoids

Students apply and practice using a visual model or an area formula to solve trapezoid problems.

Day 14

Problem Solving Polygon Puzzle

A five-piece puzzle can be formed into two rectangles, two parallelograms, one triangle, and three trapezoids. It provides an opportunity to review and summarize the meaning of the formulas and see their interrelatedness.

Day 15

Assessment Geoboard Designs

Some very interesting and complex shapes are made by combining polygons on dot paper. The problems can be solved visually or by using formulas.
**How is finding the area of a parallelogram different from finding the area of a rectangle?**

It is crucial to be able to differentiate among a side, the base, and the height. A parallelogram has four sides. The base is one of these sides. The height is only a side of a parallelogram when the parallelogram is a rectangle. Using the Measuring Pad, students focus on these differences. By cutting up a parallelogram and reforming it into a rectangle, one discovers the relationship of the two and the similarity of finding area of both.

**Materials**
- Scissors
- Parallelograms
- Measuring Pad

**Investigation**

1. Use the Measuring Pad to find and record the lengths of the sides of each parallelogram. (Round to the nearest whole centimeter, if necessary.) Determine and record the perimeter of each parallelogram.

2. Make the long side of each parallelogram the base. Measure and record the length of the base and the height. Make the short side of each parallelogram the base. Measure and record the length of the base and the height.

3. Cut a dotted line marking one of the heights of the parallelogram. Cut different lines on each pair of congruent parallelograms. Make a rectangle using the two pieces from each parallelogram. Determine and record the area of each parallelogram by finding the area of its two pieces making the rectangle.

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<thead>
<tr>
<th>Parallelogram</th>
<th>Short Side</th>
<th>Long Side</th>
<th>Perimeter (cm)</th>
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<tbody>
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<td>A</td>
<td>8</td>
<td>12</td>
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<tr>
<td>B</td>
<td>9</td>
<td>12</td>
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<td>C</td>
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<tr>
<th>Base (cm)</th>
<th>Height (cm)</th>
<th>Area (cm²)</th>
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<td>12</td>
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**Comparison**

*Students reinforce their understanding of the relationship of a parallelogram to a rectangle when finding perimeter and area.*

**Comics**

*It is crucial to be able to differentiate among a side, the base, and the height.*
Your group will need two of each parallelogram.

Cut out the parallelograms along the bold lines.
How is finding the area of a parallelogram different from finding the area of a rectangle?

1. Use the Measuring Pad to find and record the lengths of the sides of each parallelogram. (Round to the nearest whole centimeter, if necessary.)

Determine and record the perimeter of each parallelogram.

2. Make the long side of each parallelogram the base. Measure and record the length of the base and the height.

Make the short side of each parallelogram the base. Measure and record the length of the base and the height.

3. Cut a dotted line marking one of the heights of the parallelogram.

Cut different lines on each pair of congruent parallelograms.

Make a rectangle using the two pieces from each parallelogram.

Determine and record the area of each parallelogram by finding the area of its two pieces making the rectangle.

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1. What two dimensions did you use to determine the perimeter?

2. How did you use those dimensions to find the perimeter?

3. What two dimensions do you use to determine the area of the rectangle made from the two pieces of the parallelogram?

4. How are the dimensions used for area different than the ones used for perimeter?

5. Does it matter which side is the base? Explain.

6. Write a formula to describe how you use these two dimensions to get the area (A) of the parallelogram.
KEEP GOING
THINGS TO LOOK FOR:

1. What do you know about the sides of a parallelogram?
2. How do you find the perimeter of a parallelogram?
3. What is the base of a parallelogram?
4. How do you find the height of a parallelogram?
5. What is the formula for finding the area of a parallelogram?

After you cut out the parallelograms at the beginning of this activity, you measured some lengths. Let’s talk about that.

Yeah, we measured the sides. We used this pad thing to measure them.

We only needed to measure two sides because the sides that are across from each other are the same length.

That’s right. Mark, for every parallelogram, the sides across from each other are not only parallel, but they are also equal in length!

Okay, how many sides do you have to measure?

After measuring, we added up the lengths of the four sides to find the perimeter.

The perimeter is how far it is around the parallelogram.

I get it, the parallelogram is taller when it’s resting on the short side, and it’s shorter when it’s resting on the long side.

We call the base it’s resting on we call that the base.

Yeah, and whatever side the parallelogram is resting on, we call that the base.

So, when parallelogram C is resting on the short side, the height is 12. Now we know how tall it is!

Like if parallelogram C is resting on the long side, then that’s the base and that’s 16. If you measure the height from that base it’s 9.

Well, yeah, that’s pretty obvious.

Wait, now the height changed. It’s not 12 anymore.

Yeah, real things change when you use a different side for the base.

Well done. Other way is fine for finding the perimeter.

The perimeter for parallelogram C was 2 times 12 plus 2 times 16.

Well, we first had to measure the heights of the parallelograms.

The next thing we did was to find the area of each of the parallelograms, right?

Yeah, we measured the sides. We used this pad, we rested the measuring pad was like two rulers to each other. It was easy to put one side of a parallelogram on the bottom ruler and measured the height of the perpendicular dotted line.

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The next thing we did was to find the area of each of the parallelograms, right?
Does that mean we have to know two different formulas for the area of a rectangle?

And I wanted to show you how that helped us find a formula for finding the area of any parallelogram.

It’s because we already knew the formula to find the area of a rectangle.

So, class, how did we do it? How did we figure out the formula for finding the area of any parallelogram?

But right now, we can use base times height to help us figure out a formula for the area of any parallelogram.

Does that mean we have to know two different formulas for the area of a rectangle?

We found out that another way to think about the formula for area of a rectangle is that it’s base times height.

What we found out was that length and width on a rectangle are the same thing as base and height. So, a rectangle has a base and height just like all the rest of the parallelograms.

If 12 is the base of the rectangle, then the height is 8, right? And the area is base times height, that’s 12 times 8 or 96.

But right now, we can use base times height to help us figure out a formula for the area of any parallelogram.

So, class, how did we do it? How did we figure out the area of any parallelogram?

And the parallelogram and the rectangle have the same area because they’re both made out of the same pieces.

Is that the formula for the area of a parallelogram?

Is it base times height for every parallelogram?

That is an excellent summary, redmond. You’ve got it!

And you can use the short side for the base or you can use the long side, right?

But for a parallelogram, the area is just base times height.

And that’s right, juana. You just have to be sure that you measure the height from that base.