Proportional Reasoning

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Background Information

Plastics, also called resins, have different properties. There are six resins which are used most commonly in packaging. These six resins have different performance characteristics, so handlers of recycled plastics are challenged to avoid the contamination that is the result of resin types mixing. In 1988, The Society of the Plastics Industry, Inc. introduced a resin coding system to be placed on the bottom of containers to help facilitate the separation and processing of plastics. The six packaging resins were each assigned a number 1-6. The number is found inside a triangle of chasing arrows.

The type of plastic used in this activity is polystyrene. Its resin code is 6 often with the resin abbreviation, PS, beneath the triangle. Polystyrene is a clear, rigid, and brittle plastic. It is used to make cups, tableware, food packages, and even furniture. When air is added, making polystyrene foam, it can be used for hot beverage cups and packaging peanuts.

One of the characteristics of polystyrene is that it has memory. Memory means that when it is heated it will reform back into its original size and shape.

Flat polystyrene will shrink consistently when heated. This makes it an interesting material on which to draw pictures and watch them shrink as they are heated. While the length and width of the plastic is reduced, the thickness increases.
Management
1. Gather clear polystyrene. The clear disposable food containers at take-out restaurants (salads) and grocery stores (baked goods) are often polystyrene. Well before the activity have students start gathering appropriate containers. Have them check that the plastics they gather have a resin code of 6. The containers can also be purchased at a restaurant supply store.
2. This activity works well if it is broken up into two periods. One to draw and make the shrink art, the second to determine if it has shrunk proportionally.
3. Two alternatives for student interpretation are provided. The first has students determine if the scale factor is consistent for all dimensions and provides an application for proportional reasoning. The second alternative has students compare many corresponding dimensions and interpret their relationship graphically to encourage algebraic thinking along with proportional reasoning. Before beginning the activity determine which alternative is most appropriate for the class.
4. Before beginning the activity, the teacher needs to try out the materials and become familiar with the procedure. If a toaster oven is being used, check that it will work; some ovens do not have enough circulation to evenly heat the plastic. A heat gun may be used in place of an oven.
5. Because the thickness of the plastic increases as it melts, it is most easily cut and holes punched before it is heated. The brittle nature of the plastic requires that sharp scissors be used in cutting. To prevent sharp edges and corners, the scissors should be held perpendicular to the plastic when cutting it out.
6. The colored designs will adhere to the plastic better if the plastic is roughed up with fine sandpaper before tracing. The designs should be shaded in. The colors will become more intense as the plastic shrinks.
7. When the plastic pieces are initially put into the 350° oven, they will curl in upon themselves and look like they are ruined. However, when thoroughly heated, they will be shrunk and lying flat. They should then be removed from the oven and allowed to cool on a flat surface.
8. Although designed as a math investigation, this activity makes a nice craft project for making holiday ornaments or key chains. If the shapes are cut out before heating they can be strung or hung as jewelry.
9. Students should be familiar with finding percents from ratios before beginning this activity.

Procedure
1. Show the students a teacher-made sample of shrink art. Have them discuss how big they think the sample was before it was heated. Discuss the Key Questions.
2. Have the students cut out a flat rectangular piece of clear polystyrene from what they collected.
3. Using the first student sheet, tell students to draw a design they want to go on their plastic within the outline provided.
4. Direct the students to transfer the design onto the plastic by placing the plastic on top of the outline and tracing the design with permanent markers or colored pencils. The design should be shaded in. Emphasize that the measurement dots need to be traced onto the plastic also. If the art is to be hung, punch a hole in the plastic before heating it.
5. After placing the plastic on a cookie sheet, put it in a 350° oven for several minutes. When the plastic pieces are flat, take them out and allow time for them to cool.

Alternative 1
1. Using their original drawing and the shrink art, direct students to measure and record the before and after dimensions to the nearest millimeter on the second student sheet. Instruct them to use the measuring dots as points of reference for finding the width, height, and diagonal lengths.
2. Have students determine the scale factor as ratio, decimal equivalent, and percent using their measurements.
3. Direct students to find the average percent scale factor from their three sets of data.
4. Have students compare their averages with other students and discuss how they could make shrink art of a specific size using their data.

Alternative 2
1. Using their original drawing and the shrink art, have the students choose a variety of dimensions in their art. Direct them to measure and record the corresponding dimensions of both the drawing and the shrink art on the third student sheet.
2. Have students find the decimal ratio describing the relationship of the shrink art to the drawing for all the dimensions.
3. Direct the students to make a scatter plot of the corresponding dimensions to see the relationship of the shrink art to the drawing.
4. Discuss how the line formed by the data relates to the relationship of the shrink art dimensions and the original art.
5. Using the data and/or the graph have the students develop an equation that describes the relationship of the shrink art dimensions and the original art’s dimensions.

Discussion
1. How much did your plastic piece shrink?
2. How close did it come to shrinking the same amount in all directions?
3. What about the ratios communicates that the plastic shrinks consistently? [all ratios are very close to the same]
4. What about the graph communicates that the plastic shrinks consistently? [forms a line]
5. Did everyone’s art shrink the same amount? Explain.
6. How could you predict the final size of art on a new piece of plastic?
7. How can you use the scale factor or ratio to predict the final size of art on a new piece of plastic before it is heated?
8. How can you use the graph to predict the final size of art on a new piece of plastic before it is heated?
9. Write an equation that can be used to predict the final size of art on a new piece of plastic before it is heated?
10. How could this shrinking plastic be used?

Extensions
1. Have students do a data analysis of the class’s average scale factors to see if they can summarize the data. They might find mean, median, mode, and range to check the consistency of change.
2. Have students make a design the size they would like their art to be in final form. Have them use their scale factor to determine how big to make the design so it will be the desired size when shrunk.
3. Have students shrink different objects made of polystyrene (cups, bowls, flatware) to see them “remember” the shape of the original form.

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Make a Drawing
a. Draw a design or picture that will fill the frame below.
b. Use colored pencils to shade in the drawing the way you want it.

Prepare a Plastic Sheet
a. Cut out a square of polystyrene to cover the frame.
b. Rough up one side of the plastic with sandpaper until it is frosted.

Transfer the Drawing
a. Place the plastic, frosted side up, on top of your design in the frame.
b. With colored pencils trace your design onto the plastic.
c. Be sure to trace the four black dots onto the plastic.

Shrinking the Plastic
a. Preheat the oven to 350° F.
b. Place the plastic on a cookie sheet.
c. Put the cookie sheet with the plastic in the oven.
d. Watch it. The plastic will curl up and then flatten out. If a corner should get stuck on itself, you may need to remove it from the oven and pull the corner away from the plastic. Return it to the oven.
e. When the plastic has been flat for 30 seconds, remove it from the oven and cool it on a flat surface.
Using the measuring marks on your original drawing and shrink art, measure the three dimensions below to the nearest millimeter. Then determine the scale factor for each dimension.

Original Measurement: ______ mm

Shrink Art Measurement: ______ mm

**Width**

Scale Factor

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Decimal</th>
<th>Percent</th>
<th>Equivalent</th>
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</thead>
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Original Measurement: ______ mm

Shrink Art Measurement: ______ mm

**Height**

Scale Factor

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Decimal</th>
<th>Percent</th>
<th>Equivalent</th>
</tr>
</thead>
</table>

Original Measurement: ______ mm

Shrink Art Measurement: ______ mm

**Diagonal**

Scale Factor

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Decimal</th>
<th>Percent</th>
<th>Equivalent</th>
</tr>
</thead>
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What was the average scale factor for your art?
<table>
<thead>
<tr>
<th>Dimension Description</th>
<th>Original (mm)</th>
<th>Shrunken (mm)</th>
<th>Ratio: shrunk/original</th>
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Average Ratio: